SIMULATION OF HEAT AND MOISTURE INDUCED STRESS AND STRAIN OF HISTORIC BUILDING MATERIALS

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ABSTRACT
Climate change can cause not only damage, but can also destroy the basic structure of historic sites, as well as their associated interior artefacts. The evaluation of combined heat and moisture induced stress and strain (HMSS) is a valuable method in which possible damage-related processes can be predicted.

This paper presents the development of one- and two-dimensional HMSS models of building materials and artefacts in the commercial finite element software COMSOL Multiphysics Version 4.1. This multiphysics software package ensures the compatibility of modelling combined physical transports, due to its flexible and adaptable platform. Validation by means of analytical, numerical and experimental solutions are also included, along with numerical simulation results.

The HMSS model was found to be an adequate predictive tool to determine possible damage-related processes in building assemblies and artefacts.

INTRODUCTION
The driving force of this study is the Climate for Culture project found within the European Union’s 7th framework program. This project attempts to face the challenges of climate change, while acknowledging the need for the preservation of cultural heritage. The effects of climate change on selected cultural heritage sites from 16 countries in Europe and North Africa are being examined over a period of five years. It is important to consider that a change in climatic conditions can cause not only damage, but can also destroy the basic structure of these sites, as well as their associated interior artefacts.

The need for establishing attainable environmental guidelines for museums is on the emerging front. As of late, there has been not only a push to reduce energy consumption and carbon emissions, but also a concern regarding the preservation of natural resources and cultural heritage (International Institute of Conservation, 2010). As such, it is of interest to gain knowledge of the past and present conditions of artefacts in order to understand causes of previous damage, predict forthcoming damage, and also aid in developing preservation guidelines for artefacts.

The development of finite element based numerical models have been deemed as valuable tools for predicting past and future behaviour of artefacts. For example, the numerical modelling of moisture induced stress fields in a lime wood cylinder subjected to changing climate conditions has been established by Jakiela et al (2007). The purpose of this model was to determine the response of the specimen to step changes in thermal and hygric conditions, as well as the maximum stress levels. Furthermore, the inclusion of varying outdoor and indoor thermal and hygric climates in such a coupled multiphysics model could yield a more realistic predictive tool for use in historical buildings. The main focus of this study was to develop numerical models of this type in the commercial finite element software COMSOL Multiphysics Version 4.1. The compatibility of the combined heat and moisture transport, along with induced mechanical stresses and strains can be ensured by the flexible and adaptable platform available in this multiphysics software package.

This paper presents the development of one- and two-dimensional coupled heat and moisture induced stress and strain (HMSS) models. The verification of these models is executed in various stages by means of analytical, numerical and experimental solutions. The numerical simulation results for a case study consisting of an artefact’s relation to a building assembly in a historic building is also included. Finally, general conclusions and recommendations about the applicability of the HMSS models are presented.

HEAT AND MOISTURE INDUCED MECHANICAL STRESS AND STRAIN MODELLING
This study primarily focuses on the development of a general model that can be adapted to simulate HMSS of various hygroscopic materials found in historical buildings subjected to variable indoor and outdoor climates.
The HMSS model was developed in two stages based on the COMSOL numerical modelling by Schellen & van Schijndel (2010). Firstly, a one-dimensional transient coupled heat and moisture (HM) model was generated and verified to ensure the numerical validity of the simulation results. Thereafter, this verified model was expanded to include the computation of the mechanical stresses and strains which are induced by the coupled thermal and hygric transport. This coupling allows the numerical simulation of the temperature and moisture profiles, as well as the resulting mechanical stresses and strains in a defined hygroscopic material.

**Heat and Moisture Transfer**

The model is based on two-dimensional conductive heat transfer according to Fourier’s Law with a moisture dependent thermal conductivity:

\[
q_c = -\lambda \nabla T = -\left( \lambda \frac{\partial T}{\partial x} , \lambda \frac{\partial T}{\partial y} \right) \tag{1}
\]

The total moisture transfer considered includes both two-dimensional vapour and liquid flow, as described respectively in Equations (2) to (4). It should be noted that the moisture properties are functions of relative humidity.

\[
g = g_v + g_t \tag{2}
\]

\[
g_v = -\delta_p \cdot p_{sat} \varphi = -\delta_p \cdot p_{sat} \left( \frac{\partial \varphi}{\partial x} \frac{\partial \varphi}{\partial y} \right) \tag{3}
\]

\[
g_t = -D_w \cdot \xi \nabla \varphi = -D_w \cdot \xi \left( \frac{\partial \varphi}{\partial x} \frac{\partial \varphi}{\partial y} \right) \tag{4}
\]

Moisture transfer was initially characterized by both partial vapour pressure and relative humidity. Through model verification however, it was shown that partial vapour pressure as a potential poses limitations particularly under non-isothermal conditions with a simultaneous step change in hygric conditions. A delayed response in the convergence of the prescribed boundary step change in both temperature and partial vapour pressure was observed in the numerical simulation results. In short, it is thought that this delayed response is caused by the fact that the heat front propagates considerably faster than the moisture front (i.e. with a factor of 5×10^3). Moreover, the relative humidity was observed to be an effective potential in this particular case. A prescribed step change in relative humidity is in fact a function of temperature, therefore being directly correlated to the non-isothermal conditions.

**Energy and Moisture Balance Equations**

The coupling of dynamic heat and moisture transports is accomplished by means of the partial differential equations (PDE) for energy and moisture balance. These relationships can be expressed by Equations (5) and (6):

\[
c_p \rho \frac{\partial T}{\partial t} = -\nabla \left( -\lambda \nabla T \right) \pm S \tag{5}
\]

\[
\xi \frac{\partial \varphi}{\partial t} = -\nabla \left( -\delta_p \cdot p_{sat} + D_w \cdot \xi \right) \nabla \varphi \pm M \tag{6}
\]

It is of particular interest in this study to observe the correlation between temperature and relative humidity, denoted as dependent variables \( T \) and \( \varphi \) respectively. As such, these two variables, along with the associated coefficients are assembled into vectors and matrices:

\[
\begin{bmatrix}
\rho c_p & 0 \\
0 & \xi
\end{bmatrix} \begin{bmatrix}
\frac{\partial T}{\partial t} \\
\frac{\partial \varphi}{\partial t}
\end{bmatrix} + \begin{bmatrix}
\lambda \\
\delta_p \cdot p_{sat} + D_w \cdot \xi
\end{bmatrix} \begin{bmatrix}
\nabla^2 T \\
\nabla \varphi
\end{bmatrix} \pm \begin{bmatrix}
S \\
M
\end{bmatrix} \tag{7}
\]

The boundary value problem is formulated with the inclusion of two natural boundary conditions:

\[
g_c = \alpha_c \cdot (T_s - T_a) \tag{8}
\]

\[
g = \beta \cdot (\varphi_s - \varphi_a) \tag{9}
\]

These prescribed convective thermal and hygric fluxes describe the interaction between the inside of the domain with the exterior conditions. The thermal and moisture transfer coefficients are assumed to be constant for interior environments and are calculated according to the wind speed for exterior environments.

**Mechanical Stress and Strain**

The HMSS model was developed as a linear elastic boundary value problem. This type of problem is based on equilibrium, compatibility and constitutive relationships of a three-dimensional volume element in a continuous body (Eschenauer et al., 1997).

The mechanical equilibrium is based on three tensor partial differential equations that can be summarized by the equation of motion such that conditions of static equilibrium are considered:

\[
\nabla \sigma + F = 0 \tag{10}
\]

Compatibility is expressed by six equations describing the small strain-displacement of a continuous body. The total strain tensor is written as:

\[
\epsilon = \frac{1}{2} [\nabla u + \nabla u^T] \tag{11}
\]

The constitutive relationships are based on the linear proportionality between stress and strain tensors as described by Hooke’s law:
\[ \sigma = C : \varepsilon \]  
(12)

The Voigt notation can be used to express the above relationship in matrix form, shown for a two-dimensional isotropic problem:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{xy}
\end{bmatrix}
\]
(13)

The C-matrix can be further expressed by means of two independent material coefficients, namely Young’s modulus, \( E \), and Poisson’s ratio, \( \nu \) (Ottosen, & Petersson, 1992). This matrix is shown in Equation (14) for a plane strain problem. In the HMSS model, Young’s modulus is a function of moisture content and Poisson’s ratio is a constant coefficient.

\[
C = \frac{E}{(1 + \nu)(1 - 2\nu)}
\begin{bmatrix}
1 - \nu & \nu & 0 \\
\nu & 1 - \nu & 0 \\
0 & 0 & 1 - 2\nu
\end{bmatrix}
\]
(14)

The Hooke-Duhamel’s law for thermo-elastic materials can be used to relate the stress and strain tensors, along with temperature and moisture. In this study, the displacement formulation is used, as it is of interest to solve for unknown displacements as per the following:

\[
\sigma = \sigma_0 + C : (\varepsilon - \varepsilon_0 - a\Delta\theta - \kappa\Delta\omega)
\]
(15)

The inclusion of the thermal and moisture strain deformations in the strain tensor, as shown in Equation (15), allow for the coupling of the HM and HMSS models. The dimensional change coefficients included in the model are constants.

The boundary value problem using the displacement formulation requires prescribed displacements as boundary conditions. The assigned boundary values are variable depending on the geometry and orientation of the domain taken in consideration.

**Model Verification**

Verification of the HM and HMSS models were executed in various stages in order to ensure the numerical validity of the simulation results. The one-dimensional stationary and transient decoupled heat and moisture transfer mechanisms were primarily verified with an analytical benchmark test. Thereafter, one-dimensional transient coupled heat and moisture transfer was verified by means of the European Provisional Standard prEN 15026. Lastly, two-dimensional transient HMSS modelling was verified using experimental and numerical simulation results derived from a study by Rachwal et al. (2010) which evaluates the response of lime wood supports in historical panel paintings to climatic changes.

**DECOUPLED HEAT AND MOISTURE MODEL VERIFICATION**

**Analytical Verification**

The analytical case considered provides individual solutions for one-dimensional transient heat and moisture transfer problems. The case incorporates the response of a homogeneous domain to a step change at one boundary having no surface resistance. The prescribed conditions for the analytical model are shown below in Figure 1 for the heat transfer problem.

The steady-state heat and moisture solutions derived from Figure 1 are described by:

\[
T(x,t) = T_s + (T_0 - T_s) \cdot \left(1 - \frac{x^2}{a^2}\right)
\]
(16)

\[
\varphi(x,t) = \varphi_0 + (\varphi_1 - \varphi_0) \cdot \left(1 - \frac{x^2}{a^2}\right)
\]
(17)

The dimensionless part \( u(x,t) \) is expressed by the Fourier number solution in Equation (18).

\[
u(x,t) = \frac{2}{\pi} \sum_{n=1}^{N} \frac{1}{n} \cdot \sin \left(\frac{n\pi x}{L}\right) e^{-n^2\pi^2 F_0(t)}
\]
(18)

The Fourier number describes the relation between the rate of conduction and the rate of storage for the homogeneous domain:

\[
F_0(t) = \frac{a \cdot \frac{t}{L^2}}
\]
(19)

Temperature and moisture profiles after 7, 30 and 365 days were numerically simulated and compared to the analytical solutions. The results are presented in Figure 2 and Figure 3.
Figure 2 The numerically simulated temperature distribution profile versus the analytical results at 7 days, 30 days and 365 days.

Figure 3 The numerically simulated relative humidity distribution profile versus the analytical results at 7 days, 30 days and 365 days.

It is observed that the numerically simulated temperature and moisture distribution profiles are consistent with the analytical solutions.

**COUPLED HEAT AND MOISTURE MODEL VERIFICATION**

**Normative Benchmark prEN15026**

The coupled model was verified with the normative benchmark test of European Provisional Standard prEN 15026. The benchmark test is based on an analytical solution for coupled thermal and hygric transport in a homogeneous semi-infinite domain that is initially in equilibrium with constant surrounding conditions of T=20°C and ϕ=50%. The domain is thereafter exposed to a step change to T=30°C and ϕ=95%. Boundary resistances and moisture sources (rain) are neglected in this benchmark. Temperature and moisture profiles after 7, 30 and 365 days are to be calculated by the model. These profile results are required to fall within +/-2.5% of the analytical solution.

Numerical simulation results for temperature and water content profiles after 7, 30 and 365 days were found to be within the limits required by the benchmark as presented in Figure 4 and Figure 5.

As such, this model appears to be a valid predictive tool to investigate the impact of variable thermal and hygric conditions on building components and artefacts found in historical buildings.

**HMSS MODEL VERIFICATION**

**Numerical and Experimental Verification**

A study by Rachwal et al. (2010) which evaluates the response of lime wood supports in historical panel paintings subjected to changing climate conditions is used to support the validity of the two-dimensional transient HMSS model. Two examples showing the historical use of wood panel paintings are depicted below in Figure 6.
One of the various tests executed in the aforementioned study is the tangential dimensional response of a lime wood specimen to two cycles of 100 hours with relative humidity step variations between 47 and 35% under a constant temperature of 24°C. Both experimental and numerical results of the dimensional response of a lime wood specimen under the test scenario are provided in the study. The numerical results were simulated by a model developed using a finite element method software package.

The numerical simulation results from the HMSS model were compared to both experimental and numerical results from the study, as shown in Figure 7. It should be noted that the critical strain of limewood was determined to be 0.002 (Rachwal et al., 2010). The numerical simulation results from the HMSS model follow a similar trend to that presented by the experimental and numerical results from the study, as shown in Figure 7. It should be noted that the critical strain of limewood was determined to be 0.002 (Rachwal et al., 2010). The deviation observed in the results is due to a lack of information regarding the relative humidity step variations. The sensitivity of the dimensional response to minimal dissimilarities in the prescribed relative humidity step variation is made evident by the two numerical simulation results from the HMSS model particularly between 125 and 200 hours. As such, additional refinement of the boundary conditions would be necessary to increase the precision of the HMSS numerical simulation results. Nevertheless, the HMSS model simulates the expected response behaviour of lime wood and is thus considered an adequate predictive tool.

CASE STUDY

A case study is used in order to demonstrate the applicability of the two-dimensional HMSS model to historical buildings.

Background

The chosen case study involves the heritage site of the Castle of Gaasbeek located in Gaasbeek, Belgium as shown in Figure 8. This building was originally constructed in 1240 AD and has been reconstructed and modified various times since then.

This Castle functions principally as a museum, but also contains offices and storage rooms. The castle’s interior contains a multitude of art treasures, such as historic oil paintings and wooden furniture and statues, which can alone be considered as a cultural heritage site (Kramer, 2011).

Climate data

The indoor climate data includes annual temperature and relative humidity measurements from the Castle of Gaasbeek. The data was retrieved from Eindhoven University of Technology’s online database entitled Physics of Monuments (Smulders & Martens, 2008). More specifically, the data used in this numerical simulation was obtained from a data collector situated at the interior of the first floor adjacent to the south east perimeter wall as per Figure 9. This location was chosen particularly because it is relatively sheltered and thus the effects of external wind driven rain can be neglected in this simulation.
The outdoor climate data used in this case study was obtained from the climate model REMO provided by the Max Planck Institute in Hamburg, Germany. This climate data spans over a period from 1950 to 2099 and is the average data for several areas near De Bilt, Netherlands (Kramer, 2011).

The climate data prescribed for both the indoor and outdoor spanned over a timeseries of one year corresponding to January 2010 to January 2011. The prescribed temperature and relative humidity boundary conditions are presented in Figure 10 and Figure 11.

Modelling
A two-dimensional HMSS model of a lime wood panel painting fixed to an exterior brick wall is considered in order to observe the interaction of artefacts with their surrounding building assemblies in historic buildings. The exterior wall assembly was defined to consist of two layers: historical solid brick (380 mm) and cement plaster (20 mm). The material properties were obtained from the Fraunhofer-Institut Bauphysik (IBP) Holzkirchen material database. The panel painting consists of a lime wood panel (10 mm) with gesso finish in accordance with typical dimensions and material properties derived from the study by Rachwal et al. (2010). A general overview of the geometry for the two-dimensional HMSS model is shown in Figure 12. It should be noted that air movement between the panel painting and the wall was neglected in this numerical simulation.

The numerical simulation was carried out by computing the HM transports occurring in both the exterior wall and panel painting according to the prescribed climatic boundary conditions, as well as the corresponding thermal and hygric material properties. Thereafter, the HMSS was calculated simply for the lime wood panel painting.

HMSS model results
A numerical simulation was computed for a time period of one year for the purpose of demonstrating the possible numerical results that can be yielded from the HMSS model. Figure 13 and Figure 14 depict the numerical simulation results for the temperature and moisture distributions in the geometry. The magnified deformation of the panel painting is also shown in these figures with a scale factor of 62.5.
Figure 13 Numerical simulation results for the temperature distribution (K) in the exterior wall and panel painting (m) after one year.

Figure 14 Numerical simulation results for the relative humidity distribution (%) in the exterior wall and panel painting (m) after one year.

Furthermore, the displacement field results occurring at the centre of the panel painting in the tangential direction are summarized in Figure 15. The displacement field is a measurement of the expansion and contraction of the wood panel as a result of its exposure to changing climate boundary conditions. The dimensional response of the panel painting appears to reach a periodic stability over the course of a one year simulation ($3.15 \times 10^7$ s) while maintaining a value below the critical strain of 0.002 for limewood.

CONCLUSIONS

One- and two-dimensional transient coupled heat and moisture induced mechanical stress and strain (HMSS) models were developed in COMSOL Multiphysics. To commence, one-dimensional transient decoupled and coupled heat and moisture (HM) models were implemented and verified by means of analytical solutions and benchmarks. Subsequently, the heat and moisture induced stress and strain mechanisms were included into the verified HM model to formulate a two-dimensional HMSS transient model. The latter model was validated through experimental and numerical results derived from a study evaluating the response of lime wood supports in historical panel paintings subjected to changing climate conditions. The verified HMSS model was lastly tailored to a case study in order to demonstrate the applicability of the model to the interaction of artefacts and building assemblies of historical buildings.

The following conclusions can be summarized for this paper:

- The coupled HM model is a valid predictive tool to investigate the impact of a change in climatic conditions on building assemblies and artefacts.
- The HMSS model is an adequate predictive tool to determine possible damage-related processes in building assemblies and artefacts.
- The verification of the HMSS model is based on the mechanical behaviour of lime wood, which in turn signifies that additional verification using other desired materials may need to be executed.
RECOMMENDATIONS
The recommendations for future applications and developments related to this paper include the following:

- The evaluation and prediction of the effects of climate change on the HMSS in building assemblies and artefacts found in historical buildings.
- The establishment of guidelines for future indoor climate demands based on the findings to prevent predicted damages to building assemblies and artefacts found in historical buildings.
- The investigation of the cause of damage-related processes in building assemblies and artefacts of heritage sites.

NOMENCLATURE

- \( q_c \) Density of convective heat flow rate (W/m²)
- \( T \) Temperature (°C, K)
- \( \lambda \) Thermal conductivity (W/m·K)
- \( g \) Density of moisture flow rate (kg/m²·s)
- \( \varphi \) Relative humidity (-)
- \( \delta_p \) Vapour permeability (kg/m·s·Pa)
- \( p_{sat} \) Saturation vapour pressure (Pa)
- \( D_w \) Liquid transport coefficient (m²/s)
- \( \kappa \) Moisture capacity (kg/m³)
- \( M \) Moisture source (kg/m³)
- \( S \) Heat source or sink (W)
- \( \rho \) Density (kg/m³)
- \( c_p \) Specific heat capacity (J/kg·K)
- \( \delta_p \) Vapour permeability (kg/m·s·Pa)
- \( \beta \) Moisture transfer coefficient (kg/m²·s)
- \( \alpha_c \) Convective heat transfer coefficient (W/m²·K)
- \( \sigma \) Stress tensor
- \( F \) Body force per unit volume
- \( \epsilon \) Strain tensor
- \( u \) Displacement vector
- \( C \) Fourth-order stiffness tensor
- \( E \) Young’s modulus (MPa)
- \( \nu \) Poisson’s ratio (-)
- \( \nabla \theta \) Temperature increment (K)
- \( \nabla \omega \) Moisture increment (%m)
- \( \kappa \) Dimensional change coefficient due to moisture increment (1/%m)
- \( \alpha \) Dimensional change coefficient due to thermal increment (1/K)
- \( F_0 \) Fourier number (-)
- \( a \) Thermal or moisture diffusivity (m²/s)
- \( L \) Length or thickness (m)

REFERENCES


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