A Simplified Method for Stationary Heat Transfer of a Hollow Core Concrete Slab Used for TABS

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ABSTRACT

Thermally activated building systems (TABS) have been an energy efficient way to improve the indoor thermal comfort. Due to the complicated structure, heat transfer prediction for a hollow core concrete used for TABS is difficult. This paper proposes a simplified method using equivalent thermal resistance for the stationary heat transfer of this kind of system. Numerical simulations are carried out to validate this method, and this method shows very small deviations from the numerical simulations. Meanwhile, this method is used to investigate the influence of the thickness of insulation on the heat transfer. The insulation with a thickness of more than 0.06 m can keep over 95 % of the heat transferred from the lower surface, which is beneficial to the radiant ceiling cooling. Finally, this method is extended to involve the effect of the pipe, and the numerical comparison results show that this method can accurately predict the effect of the pipe.

KEYWORDS

Thermally activated building systems (TABS), Hollow core slab, Equivalent thermal resistance, Simplified method, Radiant cooling

INTRODUCTION

TABS are beneficial to optimize the energy use and ensure good indoor thermal environment. In practical application, hollow core concrete slabs presented in Figure 1 are probably used with TABS. For this kind of system, the heat transfer process is more complicated compared to the commonly-used homogeneous concrete slab. Meanwhile, the predictions of surface heat flow and surface temperature are crucial for the evaluation of cooling/heating capacity and condensation risk. Therefore, an accurate evaluation method for this system is indispensable.

In the past years, several studies have focused on the heat transfer of hollow core concrete slabs. Sodha et al. (1980) and Sodha et al. (1981) conducted theoretical studies to investigate the thermal performance of the hollow concrete slab. Another similar theoretical study can be found in (Gandhidasan and Ramamurthy 1985). All of them were carried out through solving the one-dimensional heat conduction equation using the corresponding boundary conditions at the interfaces. However, in their studies the air

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cavity was considered as a regular and uniform layer, thus thermal behaviors within the hollow core concrete element as presented in Figure 1 could not be sufficiently predicted in their models. Zhang and Wachenfeldt (2009) presented finite element analyses using COMSOL to study the heat transfer and the heat storing capacity of concrete elements with air cavities. For a hollow core slab element similar as that in Figure 1, Pomianowski et al. (2011) investigated various heat transfer mechanisms within the air void in the slab and validated the simplified model used in the whole building simulation tool-BSim (Wittchen et al. 2011). One-dimensional model using the equivalent thermal conductivity was involved in their studies, but no activated pipes were considered. The early analytical solution for the heat transfer of a uniform layer with activated water pipes was presented by Koschenz and Lehmann (2000), which is only suitable for the homogeneous elements.

This study will focus on the simplification of the hollow core into a homogeneous layer using the equivalent thermal resistance. Based on the analytical solution for the homogeneous element by Koschenz and Lehmann (2000), this simplified method is capable of involving the effect of hollow core and the effect of pipe, and predicting the uneven temperature and heat flow distributions of the slab surfaces.

PHYSICAL MODEL
The original geometry of the hollow core concrete slab is shown in Figure 1. The core in the middle is an irregular circle with the maximum height of 0.13 m and maximum width of 0.117 m. The water pipes are located at the depth of 0.014 m to the lower surface of the slab, with a diameter of 0.02 m, a pipe thickness of 0.002 m and a pipe distance of 0.157 m. The water pipes are very close to the lower surface, resulting in a faster thermal response.

![Figure 1. Geometry of hollow core concrete slab with embedded water pipes](image)

Thermal properties of the concrete are listed as follows: thermal conductivity $\lambda=1.8$ W/(m K), density $\rho=2300$ kg/m$^3$, specific heat capacity $C_p=1000$ J/(kg K). Thermal properties of the pipe are listed as follows: $\lambda=0.5$ W/(m K), $\rho=1000$ kg/m$^3$, $C_p=800$ J/(kg K). Due to the symmetrical heat transfer in the slab, only one unit as shown in the magenta rectangle in Figure 1 is investigated. In this study, this system is only considered for the radiant ceiling cooling.

EQUIVALENT SIMPLIFICATION OF HOLLOW CORE CONCRETE SLAB
Heat transfer of hollow core concrete slab is three-dimensional and complicated. To simplify the calculation, some equivalent methods are used. In BSim (Wittchen et al. 2011), the concrete layer with air voids is corrected by virtual values of thermal conductivity and density, as shown in Figure 2. The corrected properties of this layer can be calculated as the following equations:

\[
\begin{align*}
\lambda_{\text{corrected}} &= \lambda_{\text{original}} \cdot \frac{p-S_0}{p} \quad (1) \\
\rho_{\text{corrected}} &= \rho_{\text{original}} \cdot \frac{p-S_0}{p} \quad (2) \\
C_p_{\text{corrected}} &= C_p_{\text{original}} \quad (3)
\end{align*}
\]

**Figure 2. Equivalent simplification of hollow core concrete slab in BSim**

According to the simplification in BSim, the hollow core structure in Figure 1 can be transformed into a three-layer construction as depicted in Figure 3.

**Figure 3. Simplification of the concrete slab studied**
Based on the equivalent thermal resistance method developed by Li et al. (2013), the multi-layer structure shown in Figure 3 can be transformed into a homogeneous single layer as shown in Figure 4 with the same thermal properties of the original layers. The dimensions are corrected as follows:

$$d_1 = S1 + S3 + S2 \cdot \frac{\lambda_{\text{original}}}{\lambda_{\text{corrected}}}$$  \hspace{1cm} (4)
$$d_2 = S4$$  \hspace{1cm} (5)

It should be noted that this equivalent simplification only keeps the same thermal resistance of this multilayer structure, which means that it is valid for the steady-state calculation. For the transient-state calculation, further study should be carried out to determine the heat capacity or the density of the structure.

For this kind of homogenous slab structure, the analytical solution was given by Koschenz et al. (2000). Therefore, analytical results for this simplification can be derived. To involve the influence of pipe on the heat transfer, this effect can be simulated as two layers with the specified thermal resistance. Therefore, Equations 4 and 5 can be changed as the follows:

$$d_1' = S1 + S3 + S2 \cdot \frac{\lambda_{\text{original}}}{\lambda_{\text{corrected}}} + \frac{2 \cdot P \cdot \delta}{\pi \cdot d_p} \cdot \frac{\lambda_{\text{original}}}{\lambda_{\text{corrected}}}$$  \hspace{1cm} (6)
$$d_2' = S4 + \frac{2 \cdot P \cdot \delta}{\pi \cdot d_p} \cdot \frac{\lambda_{\text{original}}}{\lambda_{\text{corrected}}}$$  \hspace{1cm} (7)

RESULTS

Validation of the simplified method
In order to validate the simplified method, results from the simplified method and
numerical simulation are compared under steady-state conditions. The surrounding temperatures keep at 23 °C, and the water temperature in the pipe is 16 °C for cooling. Heat transfer coefficients (HTCs) with a range of 4-20 W/(m² K) are evaluated.

Table 1 shows the average heat flow and average surface temperature for the studied slab using the simplified method and CFD simulation. The lower surface temperatures of the slab are very close using both methods, whereas the upper surface temperature by simplified method is a little bit lower than that from CFD simulation. Additionally, the differences of heat flow from both surfaces are within 5%.

Table 2 shows the maximum and minimum surface temperatures of the slab. It can be seen that the temperatures from both methods have a very small deviation. The simplified method has the uniform temperature for the upper surface, while the CFD simulation shows a temperature difference of 0.2 °C between the maximum and minimum temperatures on the upper surface.

Figure 5 shows the surface temperature distributions along the pipe distance. Both methods give the nearly same distribution for the lower surface temperature, and the only difference is the upper surface temperature. The reason for this deviation is that the simplified method transforms all different layers into one homogenous layer, but the hollow core actually exists and influences the uneven heat transfer in the upper part of the slab. Due to the pipes are closer to the lower surface, the pipes have small impacts on the heat transfer at the upper surface. If other layers like the insulation are considered, this
impact would be neglected since the heat flow from the upper surface will be extremely low.

**Figure 5. Comparison of surface temperatures (h=8 W/(m² K))**

**Figure 6. Proportion of heat flow from the lower surface**

**Influence of the thickness of insulation**

In the practical application of radiant ceiling cooling, the upper side of the slab structure probably involves the insulation layer to reduce the heat flow from the water pipes to the upper zone. Based on the simplified method, the insulation layer can also be transformed into the homogenous slab structure. Thermal properties of the insulation used in this study are listed as follows: \( \lambda = 0.04 \text{ W/(m K)} \), \( \rho = 400 \text{ kg/m}^3 \), \( \text{Cp} = 400 \text{ J/(kg K)} \).
In this section, the thickness of the insulation varies from 0 to 0.1 m, and the proportion of heat flow from the lower surface of slab is recorded. When the insulation layer is considered, equivalent calculation is carried out as given in Table 3.

**Table 3. Equivalent calculation considering the insulation layer**

<table>
<thead>
<tr>
<th>Thickness of insulation (m)</th>
<th>0</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent d1 (m)</td>
<td>0.4</td>
<td>0.85</td>
<td>1.30</td>
<td>1.75</td>
<td>2.20</td>
<td>2.65</td>
<td>3.10</td>
<td>3.55</td>
<td>4.00</td>
<td>4.45</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Figure 6 depicts the proportion of heat flow from the lower surface of the slab under different HTCs. It can be seen that for the cooling ceiling with a heat transfer coefficient of 11.0 W/(m² K), if the proportion of heat flow from the lower surface needs to be kept above 95%, the insulation with a thickness higher than 0.06 m is indispensable.

**Influence of the pipe**

To consider the effect of pipe, this simplified method is further investigated and compared to CFD simulations. Table 4 shows the results from both methods considering the effect of pipe. Comparing results in Table 4 with that in Table 1, the pipe has a certain effect on the total heat transfer of this kind of element. In Table 4 a deviation of the heat flow from the upper surface exists but the difference of heat flow from the lower surface is very small with an error lower than 5% for low HTCs. The surface temperature has an error of 0.2 °C on the upper surface and an error of 0.1-0.2 °C on the lower surface. Generally, the simplified method is capable of sufficiently predicting the influence of pipe.

**Table 4. Heat transfer considering the influence of pipe**

<table>
<thead>
<tr>
<th>HTCs (W/(m² K))</th>
<th>Simplified method</th>
<th>CFD simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q₁ (W/m²)</td>
<td>q₂ (W/m²)</td>
</tr>
<tr>
<td>4</td>
<td>13.0</td>
<td>23.1</td>
</tr>
<tr>
<td>8</td>
<td>16.6</td>
<td>39.9</td>
</tr>
<tr>
<td>12</td>
<td>18.0</td>
<td>52.8</td>
</tr>
<tr>
<td>16</td>
<td>18.7</td>
<td>63.1</td>
</tr>
<tr>
<td>20</td>
<td>19.0</td>
<td>71.4</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

In this paper, a simplified method using the equivalent thermal resistance is proposed for the heat transfer predictions of the hollow core concrete slab with TABS. This method is capable of transforming layers with different thermal parameters into one homogeneous layer as well as involving the influence of pipe on the heat transfer of this kind of system. The method has the same accuracy as the CFD simulation under steady-state conditions. Due to the characteristics of accurate and fast, this method would be beneficial to the heat transfer evaluation of this kind of TABS.

**REFERENCES**


Wittchen, K.B., Johnsen, K., and Grau, K. 2011. BSim user’s guide, Danish Building Research Institute, Denmark.